Keeping Vague Score

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1 Tolerance & Indeterminacy

Tolerance	$\forall x_i \forall x_j((F(x_i) \wedge x_i \sim x_j) \rightarrow F(x_j))$
Limits	$\exists x_i \exists x_j(F(x_i) \wedge \neg F(x_j))$
Continuity	$\forall x_i \forall x_j \exists x_k ... \exists x_{k'}(x_i \sim x_k \wedge ... \wedge x_{k'} \sim x_j)$
Sharp Cutoffs	$\exists x_i \exists x_j(F(x_i) \wedge \neg F(x_j) \wedge x_i \sim x_j)$

Bivalence	$\mathsf{T}(\phi) \vee \mathsf{F}(\phi)$
Transparency	$\phi \leftrightarrow \mathsf{T}(\phi)$
Polarity	$\neg\phi \leftrightarrow \mathsf{F}(\phi)$

2 Context

Definition (Models). A model, $\mathcal{M}$ is a triple comprising of: (i) a linearly ordered domain of objects ($\mathcal{D}_{\mathcal{M}}$); (ii) a symmetric, reflexive relation of marginal variance ($\approx_{\mathcal{M}}$);[1] and (iii) an interpretation function ($[\![\cdot]\!]$).

Definition (Contexts). A context, $c = \langle c^+, c^- \rangle \in \mathcal{D} \times \mathcal{D}$, is a pair of subsets of $\mathcal{D}$ satisfying (i) non-triviality, (ii) convexity and (iii) coherence.

Non-Triviality	$c^+ \neq \varnothing$ and $c^- \neq \varnothing$.
Convexity	If $d \in c^+$ and $d' \geq d$, then $d' \in c^+$; and
	If $d \in c^-$ and $d \geq d'$, then $d' \in c^-$.
Coherence	If $d \in c^+$ and $d' \in c^-$, then $d \not\approx d'$.

Definition (Extension). $c \preccurlyeq c'$ (c' extends c) iff $c^+ \subseteq c'^+$ and $c^- \subseteq c'^-$.

Definition (Update). $c + \mathrm{C} = Min\{c' \in \mathrm{C} | c \preccurlyeq c'\}$.

3 Validity

Definition (Support). $c \models_g \phi$ iff for all c': if $c \preccurlyeq c'$, then $c' \notin [\![\phi]\!]^-_g$.

Definition (Validity). $\phi_i, ..., \phi_j \models \psi$ iff for all c, g: $c + ([\![\phi_i]\!]^+_g \cap ... \cap [\![\phi_j]\!]^+_g) \models_g \psi$

[1]NB: $\approx$ is subject to two further constraints: first, if $d \approx d'$ and $d \geq d'' \geq d'$, then $d \approx d''$ and $d'' \approx d'$. Second, any pair of objects in the domain are related by the ancestral of $\approx$.

4 Semantics

Definition (Semantics).

i. $c \in [\![F(x_i)]\!]_g^+$ *iff* $g(x_i) \in c^+$
$c \in [\![F(x_i)]\!]_g^-$ *iff* $g(x_i) \in c^-$

ii. $c \in [\![x_i \sim x_j]\!]_g^+$ *iff* $g(x_i) \approx g(x_j)$
$c \in [\![x_i \sim x_j]\!]_g^-$ *iff* $g(x_i) \not\approx g(x_j)$

iii. $c \in [\![\neg\phi]\!]_g^+$ *iff* $c \in [\![\phi]\!]_g^-$
$c \in [\![\neg\phi]\!]_g^-$ *iff* $c \in [\![\phi]\!]_g^+$

iv. $c \in [\![\phi \wedge \psi]\!]_g^+$ *iff* $c \in [\![\phi]\!]_g^+ \cap [\![\psi]\!]_g^+$
$c \in [\![\phi \wedge \psi]\!]_g^-$ *iff* $c \in [\![\phi]\!]_g^- \cup [\![\psi]\!]_g^-$

v. $c \in [\![\phi \rightarrow \psi]\!]_g^+$ *iff* $c + [\![\phi]\!]_g^+ \models_g \psi$.
$c \in [\![\phi \rightarrow \psi]\!]_g^-$ *iff* $c + [\![\phi]\!]_g^+ \not\models_g \psi$.

vi. $c \in [\![\forall x_i \phi]\!]_g^+$ *iff* $\forall g' \in g[x_i] : c \models_{g'} \phi$
$c \in [\![\forall x_i \phi]\!]_g^-$ *iff* $\exists g' \in g[x_i] : c \not\models_{g'} \phi$

vii. $c \in [\![\mathsf{T}(\phi)]\!]_g^+$ *iff* $c \in [\![\phi]\!]_g^+$
$c \in [\![\mathsf{T}(\phi)]\!]_g^-$ *iff* $c \notin [\![\phi]\!]_g^+$

viii. $c \in [\![\mathsf{F}(\phi)]\!]_g^+$ *iff* $c \in [\![\phi]\!]_g^-$
$c \in [\![\mathsf{F}(\phi)]\!]_g^-$ *iff* $c \notin [\![\phi]\!]_g^-$